2-1 RIGID MOTION – **Recognizing Isometry, Isometries on the Coordinate Plane**

Transformation: **an operation that maps an original geometric figure, the PREIMAGE, onto a new figure called the IMAGE. A transformation can change the position, size, or shape of a figure.**

A transformation can be noted using an arrow. The transformation statement $\triangle ABC \rightarrow \triangle XYZ$ tells you that A is mapped to X, B is mapped to Y, and C is mapped to Z.

PREIMAGE (after)

(before)

$\triangle ABC \rightarrow \triangle XYZ$

"$\triangle ABC$ is mapped to $\triangle XYZ$"

$A \rightarrow X$

$B \rightarrow Y$

$C \rightarrow Z$

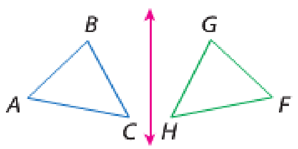
RIGID MOTION (AKA CONGRUENCE TRANSFORMATION, AKA ISOMETRY):

a transformation where the position of the image may differ from that of the preimage, but the two figures remain congruent.

KeyConcept Reflections, Translations, and Rotations

A **reflection** or *flip* is a transformation over a line called the *line of reflection.* Each point of the preimage and its image are the same distance from the line of reflection.

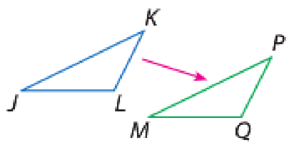
Example



$\triangle ABC \longrightarrow \triangle FGH$

A **translation** or *slide* is a transformation that moves all points of the original figure the same distance in the same direction.

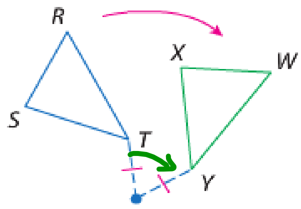
Example



$\triangle JKL \longrightarrow \triangle MPQ$

A **rotation** or *turn* is a transformation around a fixed point called the *center of rotation*, through a specific angle, and in a specific direction. Each point of the original figure and its image are the same distance from the center.

Example

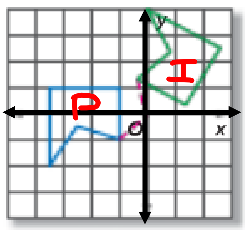


$\triangle RST \longrightarrow \triangle WXY$

ex. 1 P=preimage I=image

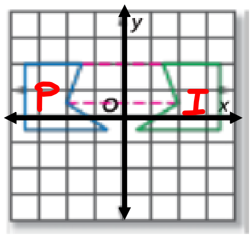
Identify the type of congruence transformation shown as a ***reflection, translation,*** **or** ***rotation.***

a.



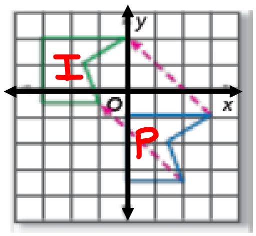
Each vertex and its image are the same distance from the origin. The angles formed by each pair of corresponding points and the origin are congruent. This is a rotation.

b.



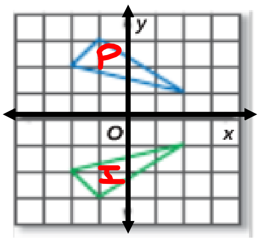
Each vertex and its image are the same distance from the *y*-axis. This is a reflection.

c.



Each vertex and its image are in the same position, just 3 units left and 3 units up. This is a translation.

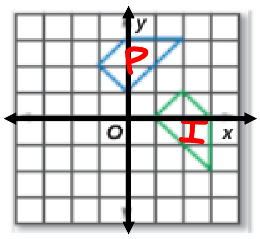
1A.



reflection

line of reflection is the x-axis.

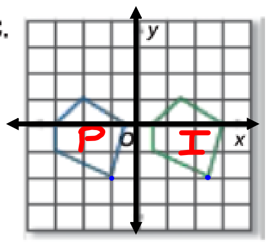
1B.



rotation

center of rotation (0,0)

1C.



translation

horizontal 3.5 units right

StudyTip

Transformations Not all transformations preserve congruence. Only transformations that do not change the size or shape of the figure are congruence transformations. You will learn more about transformations in Chapter 9.

Do you know the name of a transformation that does NOT preserve congruence?

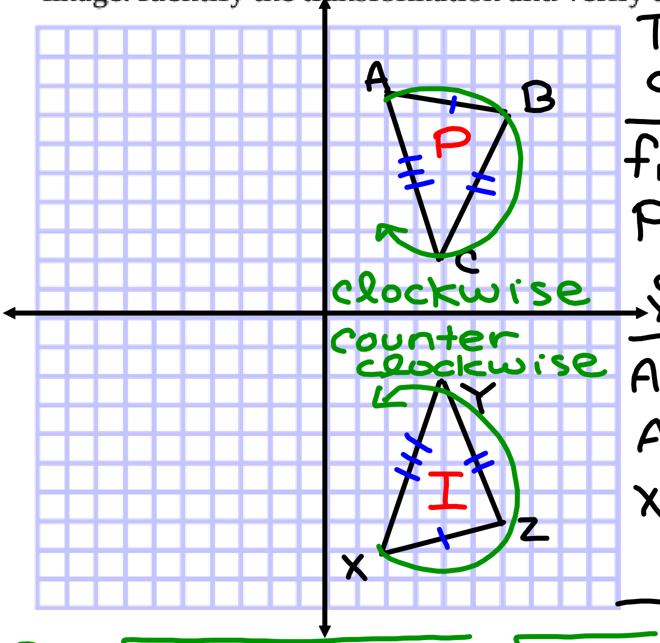
dilation

Congruence transformations will become extremely important in the study of geometry this year. We will be using them to help us prove that some objects are congruent to each other. One way to prove that two triangles are congruent to each other is by showing that all three sides of one triangle are congruent to the three corresponding sides of another triangle. We refer to this congruence relationship as **SIDE-SIDE-SIDE (SSS)**.

We can verify that reflections, rotations, and translations of triangles produce congruent triangles using the SSS relationship.

ex. 2

Triangle XZY with vertices $X(2, -8)$, $Z(6, -7)$, and $Y(4, -2)$ is a transformation of $\triangle ABC$ with vertices $A(2, 8)$, $B(6, 7)$, and $C(4, 2)$. Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.



This is a reflection over the x-axis.

find the lengths of all 3 pairs of corresponding sides.

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$AB=\sqrt{(6-2)^2+(7-8)^2}=\sqrt{16+1}$

$AB=\sqrt{17}$ $XZ=\sqrt{17}$

$XZ=\sqrt{(6-2)^2+(-7+8)^2}=\sqrt{16+1}$

$\overline{AB}\cong\overline{XZ}$

$BC=\sqrt{(4-6)^2+(2-7)^2}=\sqrt{4+25}$

$BC=\sqrt{29}$ $ZY=\sqrt{29}$

$ZY=\sqrt{(4-6)^2+(-2+7)^2}=\sqrt{4+25}$

$\overline{BC}\cong\overline{ZY}$

$AC=\sqrt{(4-2)^2+(2-8)^2}=\sqrt{4+36}$

$AC=\sqrt{40}$ $XY=\sqrt{40}$

$XY=\sqrt{(4-2)^2+(-2+8)^2}=\sqrt{4+36}$

$\overline{AC}\cong\overline{XY}$

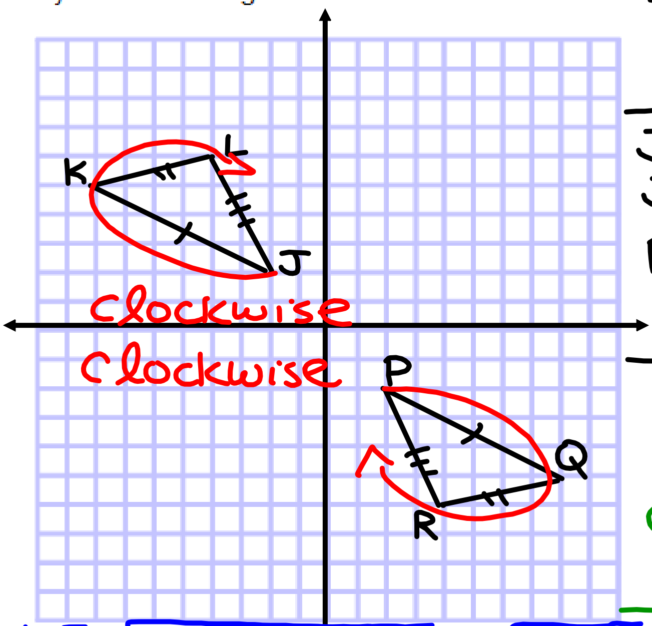
Since all 3 pairs of corresponding sides are congruent, $\triangle ABC\cong\triangle XZY$ by SSS.

Remember, an ISOMETRY preserves congruence. However, there are two types of isometries. A *direct isometry* also preserves orientation (order of lettering), while an *indirect isometry* changes this order, such as from clockwise to counterclockwise.

What type of isometry did we see in ex. 2? indirect

ex. 3

Triangle JKL with vertices J(-2, 2), K(-8, 5) and L(-4, 6) is a transformation of △PQR with vertices P(2, -2), Q(8, -5), and R(4, -6). Graph the original figure and its image. Identify the transformation and verify that it is a congruence transformation.



This is a rotation of 180°.

$JK=\sqrt{(-8+2)^2+(5-2)^2}=\sqrt{36+9}$

$JK=\sqrt{45}$ $PQ=\sqrt{45}$

$PQ=\sqrt{(8-2)^2+(-5+2)^2}=\sqrt{36+9}$

$\overline{JK}\cong\overline{PQ}$

$KL=\sqrt{(-4+8)^2+(6-5)^2}=\sqrt{16+1}$

$KL=\sqrt{17}$ $Q=\sqrt{17}$

$QR=\sqrt{(4-8)^2+(-6+5)^2}=\sqrt{16+1}$

$\overline{KL}\cong\overline{QR}$

$LJ=\sqrt{(-4+2)^2+(6-2)^2}=\sqrt{4+16}=\sqrt{20}$

$RP=\sqrt{(4-2)^2+(-6+2)^2}=\sqrt{4+16}=\sqrt{20}$

$\overline{LJ}\cong\overline{RP}$ SSS

What type of isometry did we see in ex. 3? direct

ex. 4

Identify the type of congruence transformation shown as a *reflection*, *rotation*, or *translation*.

translation

reflection